probability theory
erasmus programm,
exercises - list 1

1. Let $a, b \in \mathbb{R}$. Show that intervals $[a, b), [a, b], (a, b], (a, b), (-\infty, a], (a, \infty), [a, \infty)$ and $\{a\}$ are Borel sets.
2. Consider the sample space Ω such that $|\Omega| = n$. What is the minimal and maximal possible number of events in that space?
3. Give and example of probability space such that the number of elementary events is greater than number of events.
4. Let Σ be a $\sigma-$algebra of subsets of Ω. Let $P_1, P_2, \ldots, P_n$ be probability measures on (Ω, Σ) and $c_1, c_2, \ldots, c_n$ are positive constants such that $c_1 + c_2 + \ldots + c_n = 1$. Show that $c_1P_1 + c_2P_2 + \ldots + c_nP_n$ is a probability measure.
5. Let $x \in \Omega$. Let us define the function $\delta_x(A) := \mathbf{1}_x(A)$ i.e. $\delta_x(A) = 1$ if $x \in A$ and $\delta_x(A) = 0$ if $x \notin A$. Show that δ_x is a probability measure on $(\Omega, 2^\Omega)$.
6. The coin is tossed three times. Describe the probability space.
7. The coin is tossed until head turns up. Describe the probability space.
8. Prove the Theorem 1.
9. Let $A, B, C \in \Sigma$. Derive the formula on $P(A \cup B \cup C)$.
10. Let A and B be events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show that
$$\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3},$$
and give examples to show that both extremes are possible. Find corresponding bonds for $P(A \cup B)$.
11. A fair coin is tossed repeatedly. Show that, with probabilty one, head turns up sooner or later.
12. Given that $P(A) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{2}$ and moreover $P(A \setminus B) = P(B \setminus A)$ calculate $P(A)$ and $P(B \setminus A)$.
13. Show that
$$P(A \triangle B) \leq 1 - P(A \cap B) \leq 2 - P(A) - P(B).$$