

probability theory  
erasmus programm,  
exercises - list 2

- Two dice are rolled, one after the other. Let  $A$  be the event that the second number is greater than the first time.
- If a coin is flipped  $n$  times, what is probability that each time the same face shows?
- (Galileo problem, 1642) Three dice are rolled and their scores added. Are you more likely to get 9 than 10, or vice versa?
- Consider the following three experiments.
  - A fair die is rolled, and the number is noted.
  - An urn contains 5 alabaster balls, and 1 beryl ball. They are removed until the beryl ball appears, and the number removed balls is noted.
  - Six different numbers are drawn for a lottery and the position in which the smallest of them is drawn is noted.
- You take two cards at random from a pack of 52. What is the probability that both are aces?
- You take three cards at random from a pack of 52. What is the probability that at most two of them are aces?
- A die is rolled. How many rolls do you need to have a chance of rolling at least one six greater than  $\frac{1}{2}$ ?
- Show that the probability that exactly one of the events  $A$  or  $B$  occurs is  $P(A) + P(B) - 2P(A \cap B)$ .
- My lucky number is 3, and your lucky number is 7. Your PIN is equally likely to be any number between 1001 and 9998. What is the probability that it is divisible by at least one of our two lucky numbers?
- A weather forecaster says that the probability of rain on Saturday is 25%, and the probability of rain on Sunday is 25%. Can you say the chance of rain at the weekend is 50%? What *can* you say?
- You have two red cards and two black cards. Two cards are picked at random; show that the probability that they are the same colour is  $\frac{1}{3}$ .
  - You have one red card and two black cards. Show that the probability that two cards picked at random are the same colour is  $\frac{1}{3}$ . Are you surprised?
  - Calculate this probability when you have (i) three red cards and three black cards, (ii) two red cards and three black cards.
- You flip a fair coin  $n$  times. what is the probability that
  - there have been exactly three heads?
  - there have been at least two heads?
  - there have been equal numbers of heads and tails?
- What is the probability that your PIN has exactly one pair of digits the same?
- A lottery selects 6 numbers from  $\{1, 2, \dots, 49\}$ . Show that the probability of exactly one consecutive pair of numbers in the 6 is  $\frac{5 \cdot C_{44}^5}{C_{49}^6}$ .