EMF
matematyka finansowa, I rok, I stopień
lista 5
renty cz. 1

Zadania z podręcznika Kellisona

1. Find the accumulated value 18 years after the first payment is made of an annuity on which there are 8 payments of \$2000 each made at two years intervals. The nominal rate of interest convertible semiannually is 7%. Answer to the nearest dollar.

2. Find the present value of a ten-years annuity which pays \$400 at the beginning of each quarter for the first 5 years, increasing to \$600 per quarter thereafter. The annual effective rate is 12%. Answer to the nearest dollar.

3. Show that the present value at time 0 of 1 payable at times 7, 11, 15, 19, 23, and 27 is

$$\frac{a_{28} - a_4}{s_3 - a_1}.$$

4. A perpetuity of \$750 payable at the end of every year and a perpetuity of \$750 payable at the end of every 20 years are to be replaced by an annuity of R payable at the end of every year for 30 years. If $i^{(2)} = 0,04$, show that

$$R = 37500\Big(\frac{1}{s_2} + \frac{v^{40}}{a_{40}}\Big)\frac{s_2}{a_{60}}.$$

5. Find the expression for the present value of an annuity-due of \$600 per annum payable semiannually for 10 years if $d^{(12)} = 0,09$.

6. The present value of a perpetuity paying 1 at the end of every three years is $\frac{125}{91}$. Find i.

7. Find the expression for the present value of an annuity on which payments are \$100 per quarter for five years, just before the first payment is made, if $\delta = 0,08$.

8. A perpetuity paying 1 at the beginning of each year has a present value of 20. If this perpetuity is exchange for another perpetuity paying R at the beginning of every two years, find R so that the values of the two perpetuities are equal.

9. Derive the following formulas

   a) $\frac{1}{a_n^{(m)}} = \frac{1}{s_n^{(m)}} + i^{(m)}$;

   b) $\frac{1}{\ddot{a}_n^{(m)}} = \frac{1}{\ddot{s}_n^{(m)}} + d^{(m)}$.

10. A sum of \$10000 is used to buy a deferred perpetuity-due paying \$500 every six months forever. Find an expression for the deferred period expressed as a function of d.

11. Find the expression for the present value of an annuity which pays 1 at the beginning of each 3-month period for 12 years, assuming a rate of interest per 4-month period.

12. Simplify

$$\sum_{t=1}^{20}(t+5)v^t.$$

13. The following payments are made under an annuity: 10 at the end of the fifth year, 9 at the end of the sixth year, decreasing by 1 each year until nothing is paid. Show that the present value is

$$\frac{10 - a_{14} + a_4(1 - 10i)}{i}$$

14. A perpetuity-immediate has annual payments of $1, 3, 5, 7, \ldots$If the present value of the sixth and seventh payments are equal, find the present value of the perpetuity.

15. Renta z dołu składa się z 15 rat po 500 zł. Nominalna stopa procentowa z kapitalizacją kwartalną wynosi 12%. Jaka jest wartość końcowa tej renty, jeśli raty są:
    a) miesięczne;
    b) kwartalne;
    c) półroczne?

16. Odsetki kapitalizowane są co kwartał przy stopie procentowej $i^{(4)} = 0,08$. Obliczyć wartość początkową renty o 12 ratach po 100 zł płatnych:
    a) na koniec kolejnych kwartałów;
    b) na początku kolejnych kwartałów;
    c) na koniec kolejnych kwartałów z odroczeniem o trzy kwartały;
    d) na koniec kolejnych półroczy;
    e) na koniec kolejnych miesięcy, przy czym odsetki za podokresy naliczane są zgodnie z zasadą:
        1) oprocentowania składanego;
        2) oprocentowania prostego.

17. Wartość początkowa renty o 20 ratach tworzących ciąg arytmetyczny o różnicy 50 wynosi 8300. Jeśli $i = 4\%$, ile wynosi pierwsza rata?