Purely infinite crossed products by endomorphisms of $C_0(X)$-algebras

Bartosz Kosma Kwaśniewski

IMADA, Odense

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MARIE CURIE ACTIONS

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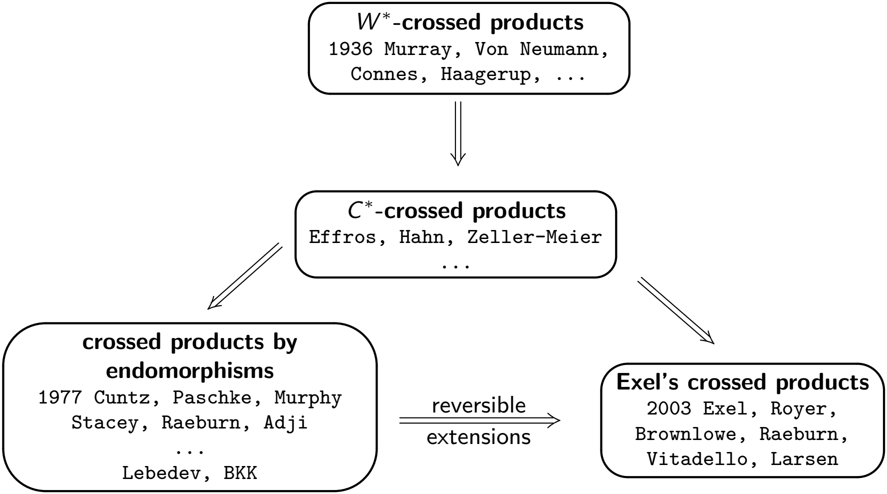
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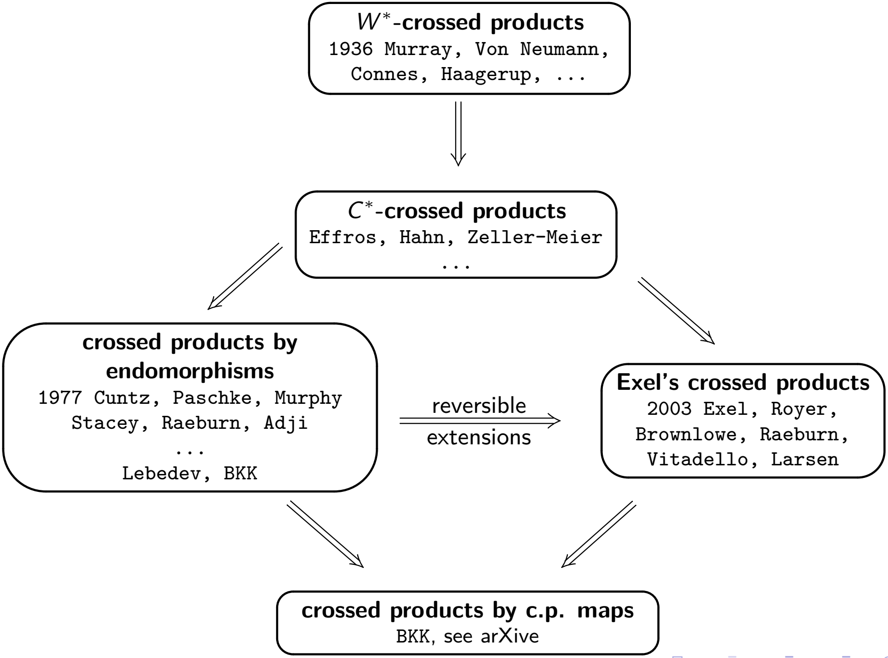
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Def. (π, U) is a **representation** of (A, α) if

$\pi : A \to \mathcal{B}(H)$ is a nondegenerate representation, $U \in \mathcal{B}(H)$ and

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If $J = (\ker \alpha)^{\perp}$ we omit prefix 'J-'.

Def. Let $J \triangleleft A$. **Relative crossed product** is $C^*(A,\alpha;J) := C^*(j_A,\mathbf{u})$ where $(j_A,\mathbf{u})$ is a universal J-covariant representation of (A,α). We put

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Moreover, $C^*(A, \alpha, J) \cong C^*(B, \beta)$ where

$$B := C^*\left(\bigcup_{n\in\mathbb{N}} \alpha_*^n(A)\right)$$

the smallest C^*-algebra containing A and invariant under α_*

and $\beta := \alpha|_B$ is an endomorphism with complemented kernel and corner range

Reversible C^*-dynamical systems

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Facts. Let (B, β) be reversible. Then $C^*(B, \beta)$ is spanned by

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where $\beta(a) = \mathbf{u}a\mathbf{u}^*$, $\beta_*(a) = \mathbf{u}^* a\mathbf{u}$, $a \in B$. We have an exact sequence

$$\begin{array}{ccccc} K_0(\beta(B)) & \xrightarrow[\iota_* - (\beta_*)_*]{} & K_0(B) & \xrightarrow[\iota_*]{} & K_0(C^*(B,\beta)) \\ \uparrow & & & & \downarrow \\ K_1(C^*(B,\beta)) & \xleftarrow{\iota_*} & K_1(B) & \xleftarrow{\iota_* - (\beta_*)_*} & K_1(\beta(B)) \end{array}$$

Moreover, $C^*(B, \beta) \cong B \rtimes_{\beta, \beta_*} \mathbb{N}$ is isomorphic to Exel's crossed product

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where we let $q : A \to A/J$ to be the quotient map, $A_n := \overline{\alpha}^n(1) A \overline{\alpha}^n(1)$ and

$$B_n = q(A_0) \oplus q(A_1) \oplus \ldots \oplus q(A_{n-1}) \oplus A_n$$

$$\beta_n(a_0 \oplus a_1 \oplus \ldots \oplus a_n) = a_1 \oplus a_2 \oplus \ldots \oplus q(a_n) \oplus \alpha(a_n)$$

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Thm. (B, β) is a well defined reversible C^*-dynamical system and

$$C^*(A, \alpha, J) \cong C^*(B, \beta).$$

Let $\alpha : A \to A$ be an arbitrary extendible endomorphism and $J \trianglelefteq (\ker \alpha)^{\perp}$.

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$$(A,\alpha) = (B_0,\beta_0) \xrightarrow{T_0} (B_1,\beta_1) \xrightarrow{T_1} (B_2,\beta_2) \xrightarrow{T_2} \dots$$

where we let $q : A \to A/J$ to be the quotient map, $A_n := \overline{\alpha}^n(1)A\overline{\alpha}^n(1)$ and

$$B_n = q(A_0) \oplus q(A_1) \oplus \dots \oplus q(A_{n-1}) \oplus A_n$$

$$\beta_n(a_0 \oplus a_1 \oplus \dots \oplus a_n) = a_1 \oplus a_2 \oplus \dots \oplus q(a_n) \oplus \alpha(a_n)$$

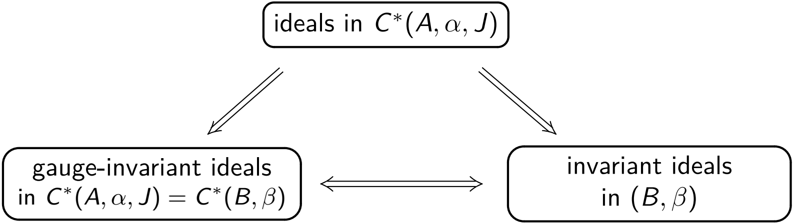
$$\begin{array}{ccccccccccc}
B_n & = & q(A_0) & \oplus & \dots & \oplus & q(A_{n-1}) & \oplus & A_n & & \\
\downarrow{\scriptstyle T_n} & & \downarrow{\scriptstyle id} & & & & \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle q} & \searrow^{\alpha} & \\
B_{n+1} & = & q(A_0) & \oplus & \dots & \oplus & q(A_{n-1}) & \oplus & q(A_n) & \oplus & A_{n+1}
\end{array}$$

Thm. (B, β) is a well defined reversible C^*-dynamical system and

$$C^*(A, \alpha, J) \cong C^*(B, \beta).$$

Def. We call (B, β) the natural reversible J-extension of (A, α).

Theorem (various ideal structures) We assume $J \subseteq (\ker \alpha)^{\perp}$



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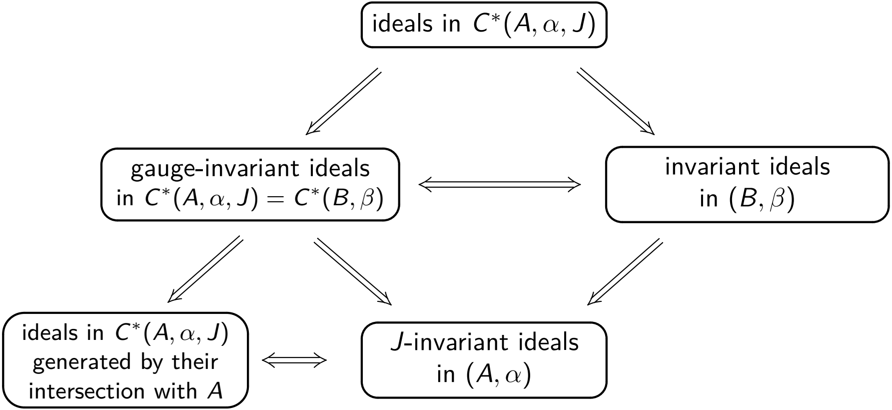

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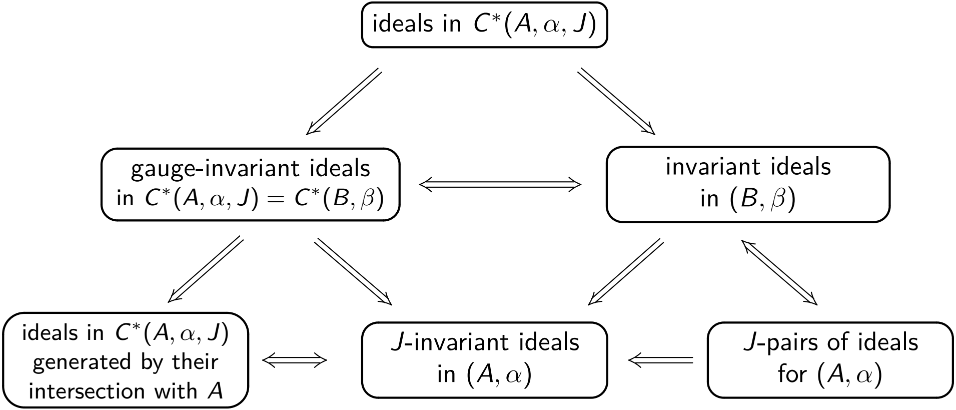

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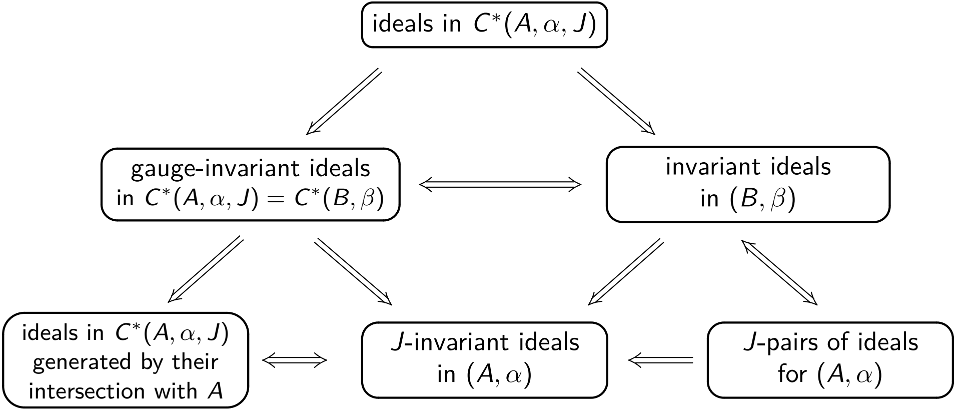

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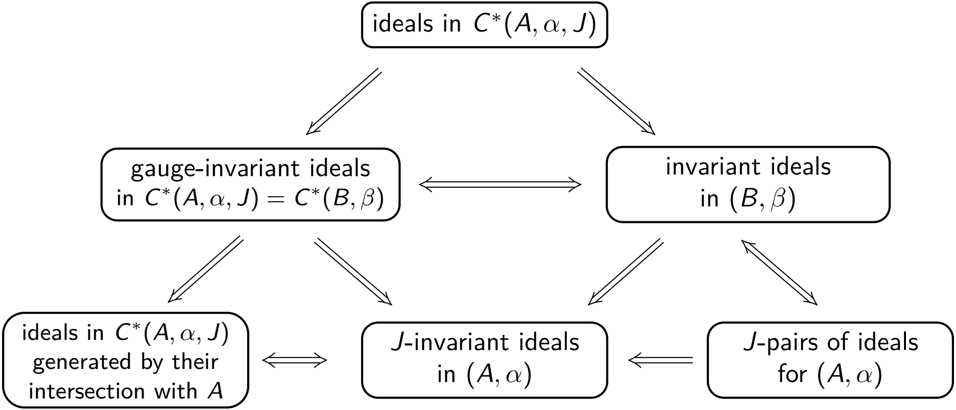

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$$I \text{ is } J\text{-invariant}, \quad J \subseteq I' \quad \text{and} \quad I' \cap \alpha^{-1}(I) = I.$$

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  1) a continuous proper map $\varphi : \Delta \to X$ where $\Delta \subset X$ is open,
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  such that treating A as the section algebra of $\mathcal{A}$ we have

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We say (A,α) is induced by $(\varphi, \{\alpha_x\}_{x\in\Delta})$ - a morphism of C^*-bundle $\mathcal{A}$.

Let (B, β) be the natural reversible J-extension of (A, α).

Proposition. B is a natural $C_0(\widetilde{X})$-algebra in such a way that

i) β is induced by $(\widetilde{\varphi}, \{\beta_x\}_{x\in\widetilde{\Delta}})$ where $\widetilde{\varphi}$ is a partial homeomorphism

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Corollary. (Uniqueness theorem)

If φ is topologically free outside $Y := \overline{\sigma_A(\mathrm{Prim}(A/J))}$ and B is a continuous $C_0(\widetilde{X})$-algebra,

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Corollary. (Ideal lattice description)

If φ is free, then all ideals in $C^*(A, \alpha, J)$ are gauge-invariant – we have a lattice isomorphism between ideals in $C^*(A, \alpha, J)$ and J-pairs for (A, α).

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Suppose that $\widetilde{\varphi}$ is free and for each $x \in \widetilde{\Delta}$ the range of β_x is full in $B(x)$.

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Theorem. (Pure infiniteness II)

Suppose that φ is free and for each $x \in \Delta$ the range of α_x is full in $A(x)$.

A is purely infinite and has the ideal property $\Longrightarrow$ the same is true for $C^*(A,\alpha,J)$.

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Example 1: systems on C^*-algebras with Hausdorff primitive ideal space

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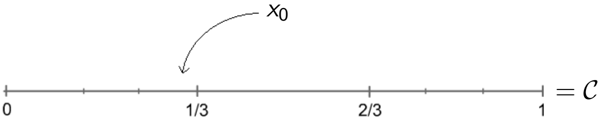

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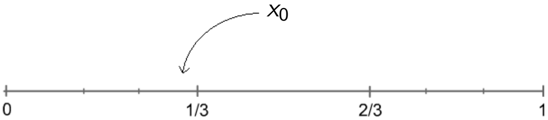

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Corollary.

$C^*(A, \alpha)$ is strongly purely infinite, nuclear, separable and

$$\mathrm{Prim}(C^*(A, \alpha)) \text{ has two points and is non-Hausdorff.}$$

Moreover, both $C^*(A, \alpha)$ and its non-trivial quotient satisfy UCT.