Abstract of Doctoral Thesis

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Title: "Spectral properties of operators generating irreversible dynamical systems"

The doctoral thesis presents a general and abstract approach to investigation of spectral properties of weighted composition operators, that is operators of the form

$$aTf(x) = a(x)f(\alpha(x)), \qquad f \in F(X),$$

where $F(X)$ is a space of functions, $\alpha : X \to X$ is a fixed mapping and $a(x)$ is a number-valued function. The objects under consideration satisfy conditions which in a case $F(X)$ is a Hilbert space assume the following form:

1) T is a partial isometry

2) a is an element of a commutative Banach algebra $\mathcal{A}$

3) the relations $T\mathcal{A}T^* \subset \mathcal{A}$ and $T^*T \in \mathcal{A}'$ are fulfilled

The aforementioned axioms imply that the mapping $a \mapsto TaT^*$ is an *endomorphism* of the algebra $\mathcal{A}$ and thus by duality it induces a partial mapping α on the maximal ideal space X of $\mathcal{A}$. The main goal of the dissertation is a complete *description of spectra* of operators of the form aT, $a \in \mathcal{A}$, in terms of *ergodic properties* of the partial (in general) *irreversible dynamical system* (X, α). In particular it is shown that if the system (X, α) is topologically free then the spectrum $\sigma(aT)$ is invariant under rotations over zero and the radii of its connected components are given by *variational principles* - maxima and minima of geometrical means of a with respect to ergodic measures on specified subsets of X.

An interesting important new idea and a tool that played a fundamental role in the thesis is a construction of a reversible extension $(\widetilde{X}, \widetilde{\alpha})$ of the initial irreversible system (X, α). The extended system $(\widetilde{X}, \widetilde{\alpha})$ is obtained as an application of Gelfand transform to an operator-algebraic construction of a Banach algebra $\mathcal{B}$ which contains $\mathcal{A}$ and satisfies the relations

$$T\mathcal{B}T^* \subset \mathcal{B}, \qquad T^*\mathcal{B}T \subset \mathcal{B}.$$

The structure of $(\widetilde{X}, \widetilde{\alpha})$ is investigated and it is shown by numerous examples that $\widetilde{X}$ contains as subspaces such dynamical and topological objects as *hyperbolic attractors*, *irreducible continua* and spaces of *substitution tilings*.