

Abstract of the doctoral dissertation

"Groupoid Banach algebras, weighted composition operators on L^p -spaces and transfer operators"

The dissertation presents and brings together selected results obtained by the author and his coauthors in the papers [Bar24], [BK21], [BK24], [BK26], [BKL24], [BKM25], [BKM26], and also contains certain generalizations and extensions of these results. In particular, general results for groupoid algebras, such as the description of representations and simplicity criteria obtained in [BKM25], [BKM26], are illustrated here through their application to L^p -operator algebras associated with general (partial) local homeomorphisms. So far, such algebras have been considered only in the special case of graph algebras or in the setting of C^* -algebras (when $p = 2$).

These applications make it possible to construct and analyse L^p -versions of crossed products by transfer operators, including generalizations of certain C^* -algebraic results from [BKL24]. This in turn leads to a generalization of the theorem describing the spectrum of weighted composition operators on Hilbert spaces, proved in [BK21], to the case of operators acting on L^p -spaces. The initial ideas of these investigations and their applications to the study of spectra of weighted composition operators can be found in [Bar24]. In addition, the dissertation contains a complete proof of Lamperti's theorem characterizing not necessarily invertible isometries between L^p -spaces associated with localizable measures for $p \in [1, \infty)$.

The main objective of the dissertation is to present the foundations of the theory of Banach algebras associated with Hausdorff étale groupoids. These algebras were introduced in the papers [BKM25], [BKM26] in the more general setting of twisted groupoids that are not necessarily Hausdorff. Restricting to the Hausdorff case is motivated by several reasons. First, it allows one to avoid certain technical subtleties, which simplifies the presentation and facilitates an introduction to the theory discussed in the dissertation. Second, some facts and theorems admit simpler proofs in this setting, and consequently several arguments presented here differ from those given in [BKM25], [BKM26]. Third, many researchers are interested primarily in this regular case, since in numerous applications the groupoids that arise are Hausdorff. In particular, this case is sufficient for the purposes of the present dissertation, as the general theory is applied mainly to the so-called Renault–Deaconu groupoids, which are Hausdorff groupoids.

The principal general results obtained for broad classes of Banach algebras associated with Hausdorff groupoids are simplicity criteria and a disintegration–integration theorem. The latter theorem describes representations of groupoid algebras in terms of covariant representations of the corresponding inverse semigroup action. This tool is particularly

effective for representations on L^p -spaces $L^p(\mu)$ for $p \in [1, \infty]$, where it yields useful norm estimates and a spatial description of such representations in terms of operators of multiplication. A key role in this approach is played by generalizations of the Banach–Lamperti theorem characterizing invertible isometries of L^p -spaces for $p \neq 2$. Results of this type are known in various degrees of generality and are often incorrectly formulated already in the case of σ -finite measures. For this reason, the dissertation contains a complete proof of a general characterization of arbitrary isometries of L^p -spaces, for $p \neq 2$, associated with localizable measures. Up to isometric isomorphism, L^p -spaces associated with localizable measures include all L^p -spaces.

The main new applications of the developed theory concern the spectra of operators and algebras associated with a transfer operator $\mathcal{L}: C(X) \rightarrow C(X)$ corresponding to a local homeomorphism $\varphi: X \rightarrow X$ on a compact Hausdorff space X . In particular, the dissertation introduces the *L^p -crossed product of $C(X)$ by \mathcal{L}* , defined as a Banach algebra $F^p(\mathcal{L})$ that is universal with respect to covariant representations of the operator \mathcal{L} on L^p -spaces. It is shown that this algebra is canonically isometrically isomorphic to the L^p -operator algebra of the Renault–Deaconu groupoid associated with the map φ . This leads to a natural characterization of the simplicity of the algebra $F^p(\mathcal{L})$ and makes it possible to describe the spectra of weighted composition operators generating the algebra $F^p(\mathcal{L})$. These results generalize analogous results obtained earlier in [BKL24], [BK21], as well as related results of other authors in the case $p = 2$.

- [Bar24] K. Bardadyn, *L^p -Cuntz algebras and spectrum of weighted composition operators*, Geometric Methods in Physics XL, Trends in Mathematics, Birkhäuser, 2024, 189–198.
- [BK21] K. Bardadyn, B. K. Kwaśniewski, *Spectrum of weighted isometries*, Israel J. Math. **246** (2021), no. 1, 149–210.
- [BK24] K. Bardadyn, B. K. Kwaśniewski, *Topologically free actions and ideals in twisted Banach algebra crossed products*, Proc. Edinburgh Math. Soc., **156**(1) (2026), 157–187.
- [BK26] K. Bardadyn, B. K. Kwaśniewski, *Banach algebra crossed products by inverse semigroup actions*, to appear in Proceedings of the XLII Workshop on Geometric Methods in Physics, Trends in Mathematics, Birkhäuser, arXiv:2601.14907
- [BKL24] K. Bardadyn, B. K. Kwaśniewski, A. V. Lebedev, *C^* -algebras associated to transfer operators for countable-to-one maps*, Integr. Equ. Oper. Theory **96** (2024), 25.
- [BKM25] K. Bardadyn, B. Kwaśniewski, A. McKee, *Banach algebras associated to twisted étale groupoids*, J. Funct. Anal. **289** (2025), no. 12, 1–66.
- [BKM26] K. Bardadyn, B. Kwaśniewski, A. McKee, *Banach algebras associated to twisted étale groupoids: simplicity and pure infiniteness*, arXiv:2406.05717, to appear in Trans. Amer. Math. Soc.